INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2016-17
Statistics - II, Midterm Examination, February 23, 2017
Answer all questions.
You may use any result stated in the class by stating it.

1. Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, respectively, from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, where $-\infty < \mu_1, \mu_2 < \infty$, $\sigma^2 > 0$.
(a) Does this model belong to the exponential family of distributions? Justify.
(b) Find minimal sufficient statistics for the unknown parameters. Is it complete?
(c) Find the MLE and UMVUE of σ^2. [15]

2.(a) Let U and V be two (jointly distributed) statistics such that U has finite variance. Show that

$$\mathrm{Var}(U) = \mathrm{Var}(\mathrm{E}(U|V)) + \mathrm{E}(\mathrm{Var}(U|V)).$$

(b) Suppose $(X_1, X_2, \ldots, X_n)$ has probability distribution P_θ, $\theta \in \Theta$. Let $\delta(X_1, X_2, \ldots, X_n)$ be an estimator of θ with finite variance. Suppose that T is sufficient for θ, and let $\delta^*(T)$, defined by $\delta^*(t) = E(\delta(X_1, X_2, \ldots, X_n)|T = t)$, be the conditional expectation of $\delta(X_1, X_2, \ldots, X_n)$ given $T = t$. Then arguing as in (a), and without applying Jensen's Inequality, prove that

$$E(\delta^*(T) - \theta)^2 \leq E(\delta(X_1, X_2, \ldots, X_n) - \theta)^2,$$

with strict inequality unless $\delta = \delta^*$ (i.e., δ is already a function of T). [15]

3. Suppose $X_1 \sim \text{Binomial}(n_1, p)$ which is independent of $X_2 \sim \text{Binomial}(n_2, p)$, where n_1 and n_2 are fixed and $0 < p < 1$.
(a) What is the conditional distribution of X_1 given $X_1 + X_2 = k$?
(b) Using (a) show that $X_1 + X_2$ is sufficient for p. [10]

4. Let $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$, and let $Y = 1$ when $X > 0$, and 0 otherwise.
(a) Find the Fisher information on λ (say, $I^{(X)}(\lambda)$ and $I^{(Y)}(\lambda)$, respectively) contained in X and Y.
(b) Compare $I^{(X)}(\lambda)$ and $I^{(Y)}(\lambda)$. [10]